Beyond Rating Curves:
Time Series Models for in-Stream Turbidity Prediction

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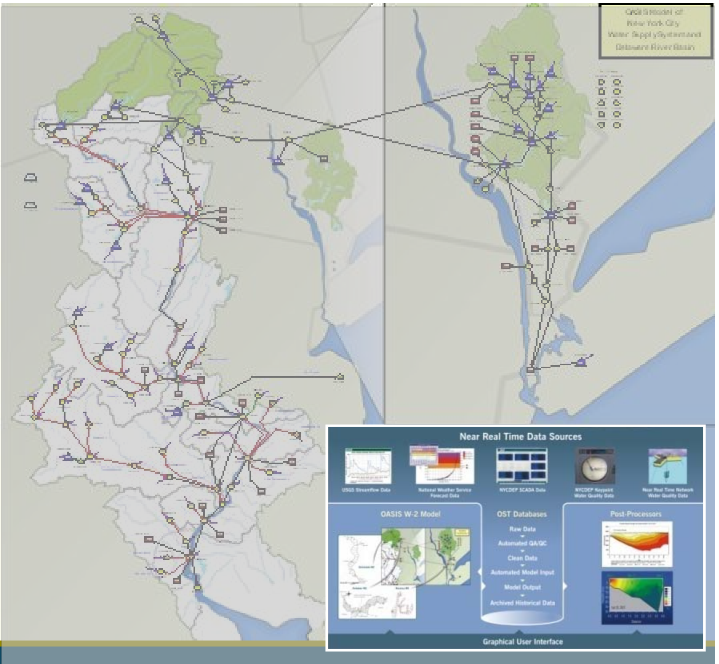

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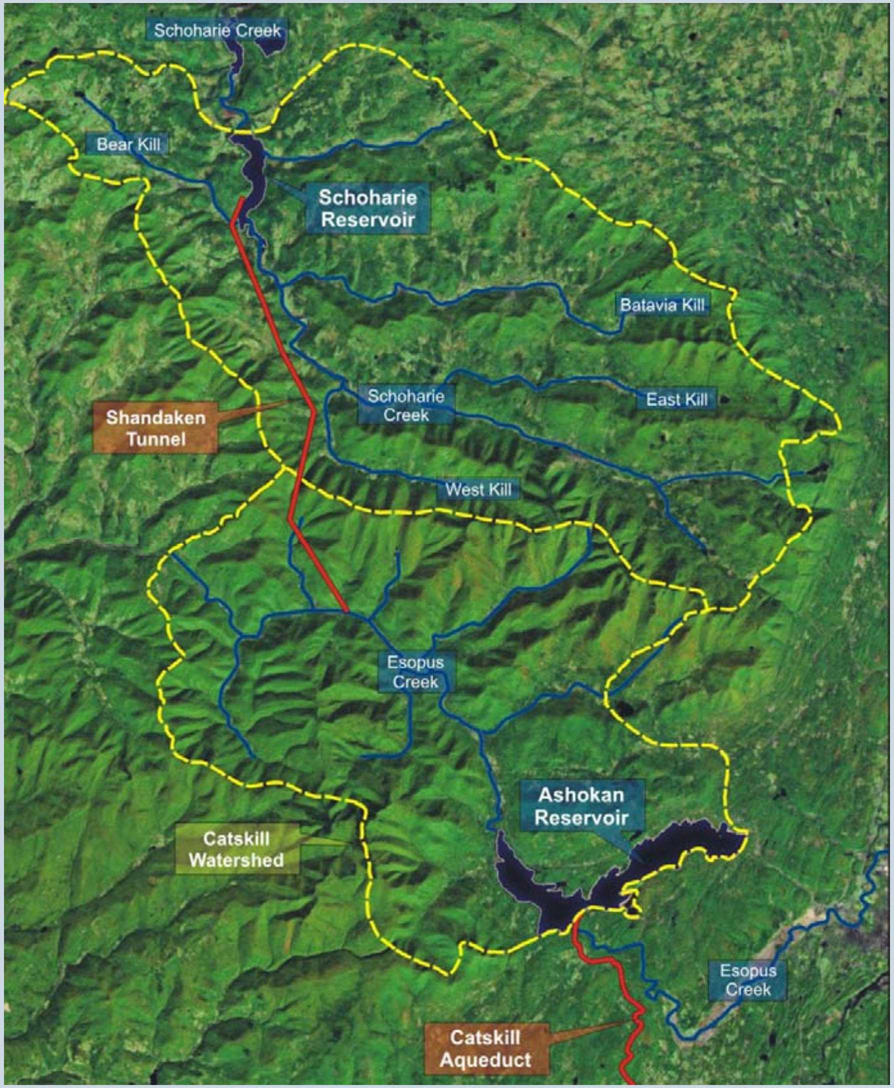
CAT-330: Catskill Turbidity Control Study

Design Services for the Development of an Operations Support Tool

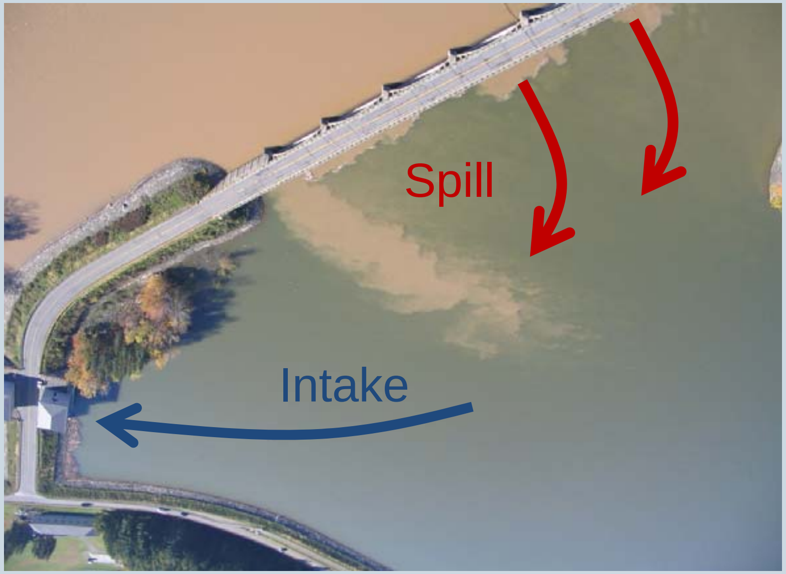
New York City's Water Supply Challenges: Water Quality

- NYC has Filtration Avoidance Determination (FAD) from EPA for Cat/Del system (2007)
- Meeting turbidity limits is an important requirement for the water supply
  - May not deliver water from Kensico over 5 NTU
- Catskill system intermittently affected by turbidity during/after high flow events



Managing Catskill Turbidity through Selective Diversions

- Per DEC, alum usage must be minimized/eliminated
- Flexibility in NYC system allows for reservoir operations adjustments for effective turb control
  - Essentially minimize turbid diversions from Ashokan
  - Most strategies have a supply reliability impact



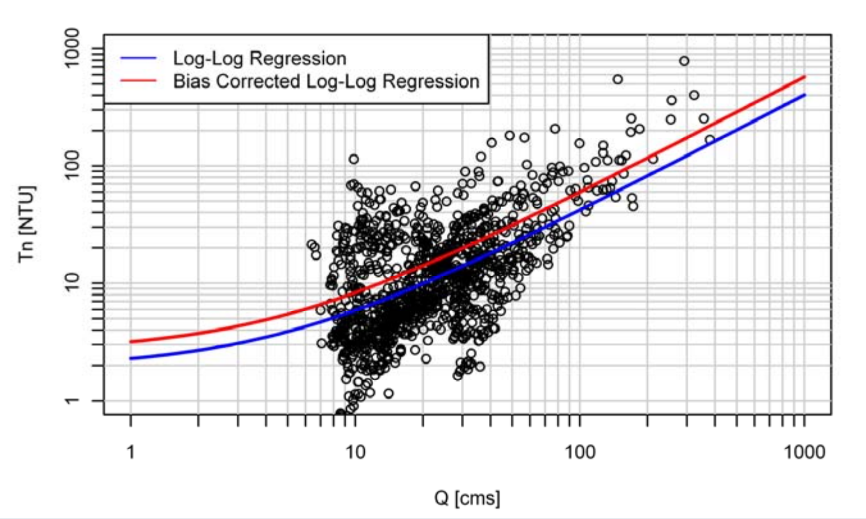
System Flexibility to Manage Turbidity

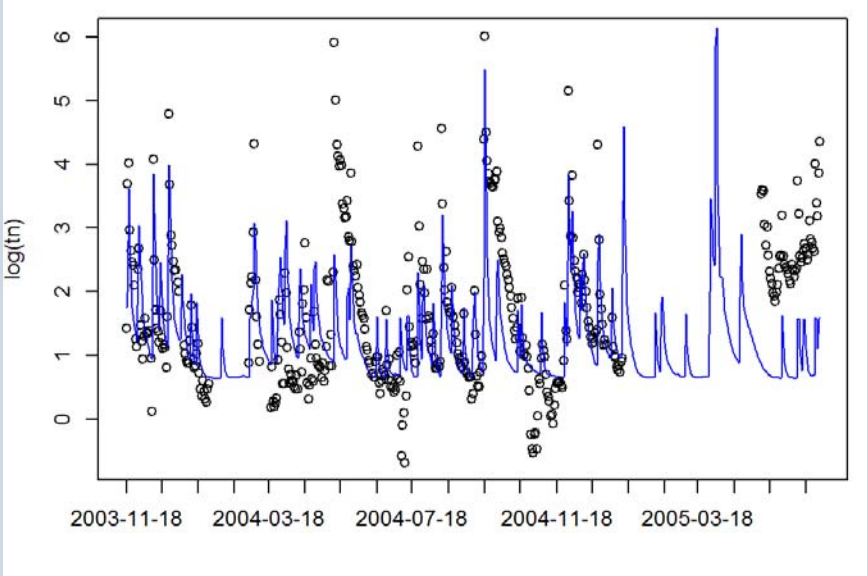
- Minimize Catskill diversions
  - Make up difference with Delaware or Croton
  - Reduce minimum flow in Catskill Aq. with stop shutters
- Increase West Basin ability to capture storm volume, allow time for turbidity to settle out, prevent spillage from West to East
  - Create void space in West Basin via preemptive diversions and/or releases from ARC

All strategies have supply reliability impacts; Important to understand how long an event will affect the system

Predicting/Estimating in-Stream Turbidity Values: Flow-Turbidity Rating Curves

- Typically linear regression of log-transformed turbidity observations and streamflow observations
  - Occasionally use squared flow as additional term in regression
- Generally poor in predicting intra and inter-event variability





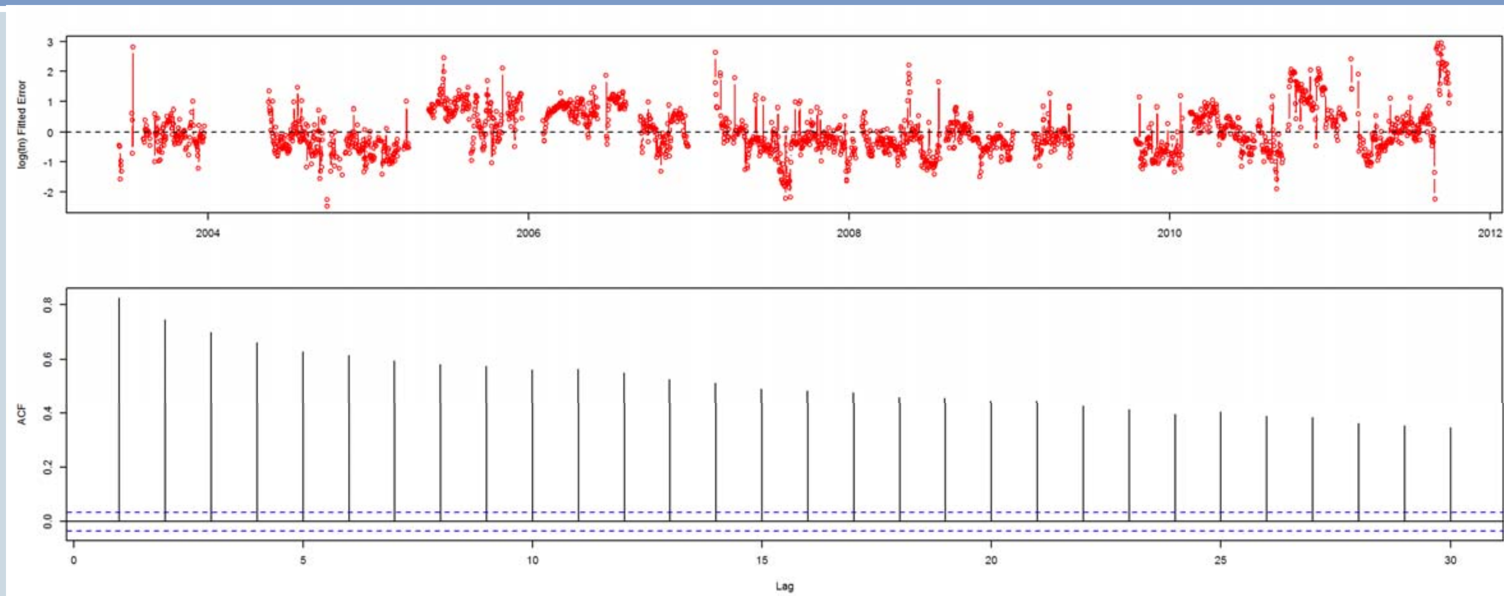
Can we do Better in Capturing intra/inter-event Variability?

- Like streamflow, turbidity is **autocorrelated** in time
  - Current (and future) observations are linearly related to recent past (time-lagged) observations
  - Recent past observations may be used to inform (predict) future observations
  - Box-Jenkins time series models may be applied instead of/as a supplement to linear regression models
    - Autoregressive (AR), Autoregressive Moving Average (ARMA), etc.

Streamflow example:

- "Last few months have been dry, this month is likely to be dry, too"
  - Physical basis is persistence in baseflow & soil moisture
- Apply this principle to turbidity observations

Introduction to Linear Regression with Autoregressive/Moving Average Errors



- Rule Curve errors (predicted – observed) are correlated in time due to persistence in streamflow and turbidity
- Correlated errors can be used as additional predictors in improving skill of rule curve estimates

Linear Regression Part:

$$Y(t) = \sum_{i=1}^{c} \beta_i X_i(t) + \mu + \epsilon(t)$$

ARMA Errors:

$$\epsilon(t) = \sum_{i=1}^{p} \phi_i[\epsilon(t-i) - \mu] + \sum_{i=1}^{q} \theta_i[\eta(t-i) - \mu]$$

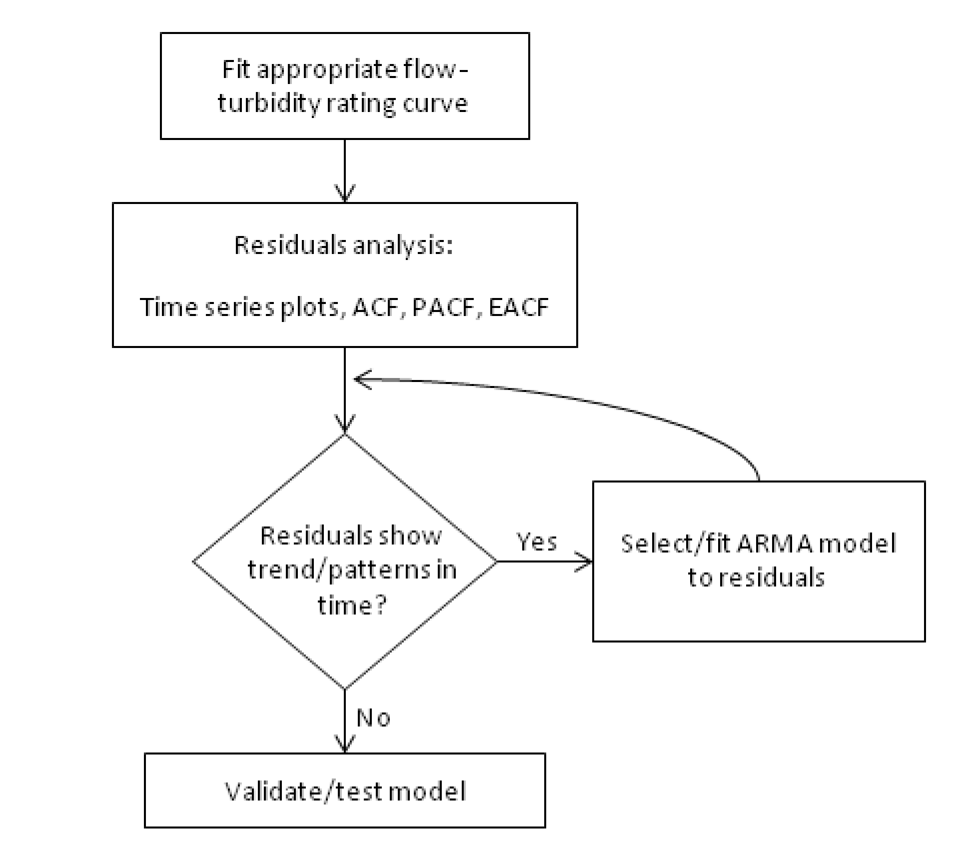
Model Design & Fitting Procedure

1) Develop Linear Regression Rule Curve

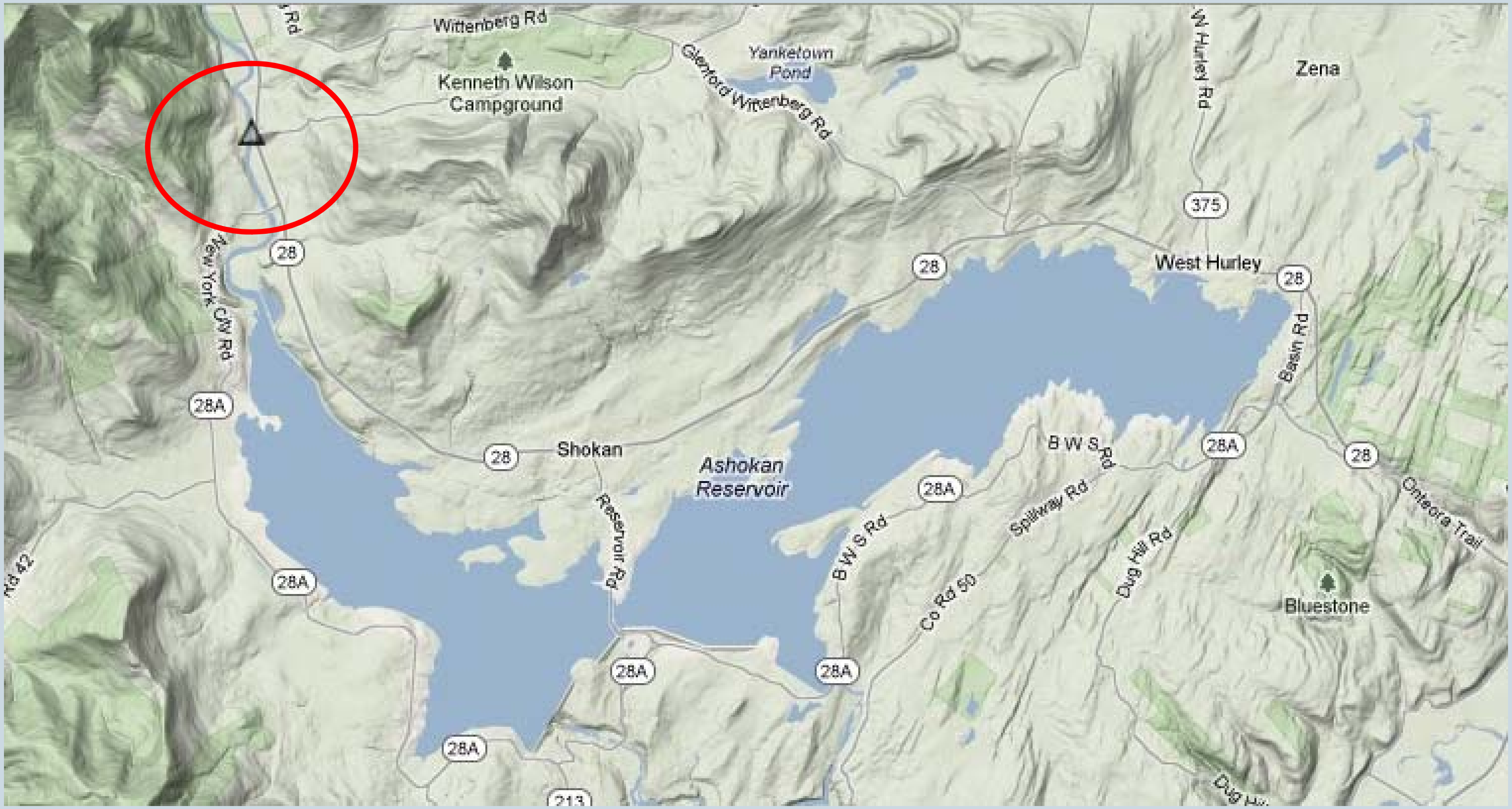
- Correlation/scatter plot analysis of potential predictors
- Fit "best" linear regression model

2) Develop Time Series Model Component

- Residuals analysis
- Determine AR and/or MA order
- Fit ARMA model to the linear regression residuals



Case Study: Esopus Creek at Coldbrook

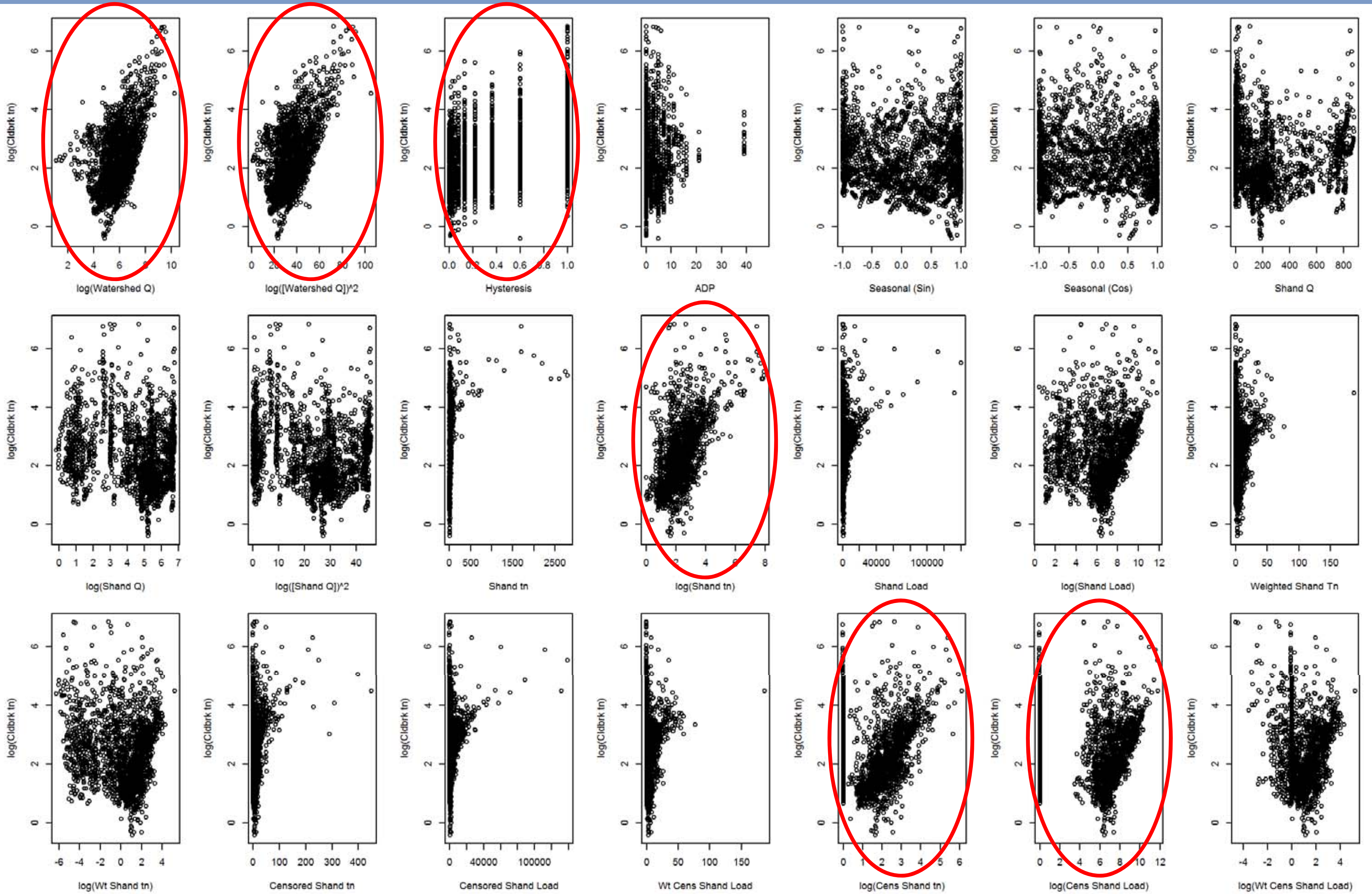


*Flow (and turbidity) from Shandaken Tunnel (Schoharie Reservoir) enters Esopus from Northwest

21 Potential Predictors Considered for Linear Regression Rule Curve

- Log(Watershed Q) and Log(Watershed Q)^2
  - "Watershed Q" = USGS gauge flow at Coldbrook – USGS observed flow at the Shandaken Tunnel outlet
  - Calculated flows below 0.1 cfs removed via linear interpolation (9 observations)
- Hysteresis effect (based on method in Hirsch, 1988)
- Antecedent dry period (ADP)
- First Fourier series of the seasonal cycle
- USGS observed Shandaken Tunnel Q, log(Shand Q), log(Shand Q)^2
- DEP Shandaken Tunnel turbidity (interpolated grab data), log(Shand tn), Shandaken Turb load, log(Shand tn load)
- Weighted Shandaken turbidity
  - (Shandaken tn * Shandaken Q) / Coldbrook Q
- "Censored" Shandaken variables (tn, tn load, weighted tn load, and log transforms)
  - Shandaken variables set to 0 if they are less than 10 mgd or are less than 20% of total Coldbrook flow

Predictor Scatter Plots



Correlation Coefficients of Significant Predictors

Predictor	Correlation Coefficient
log(Watershed Q)	0.513
log(Watershed Q)^2	0.562
Hysteresis	0.426
log(Cens. Shand. Tn)	0.157
log(Cens. Shand. Load)	-0.098

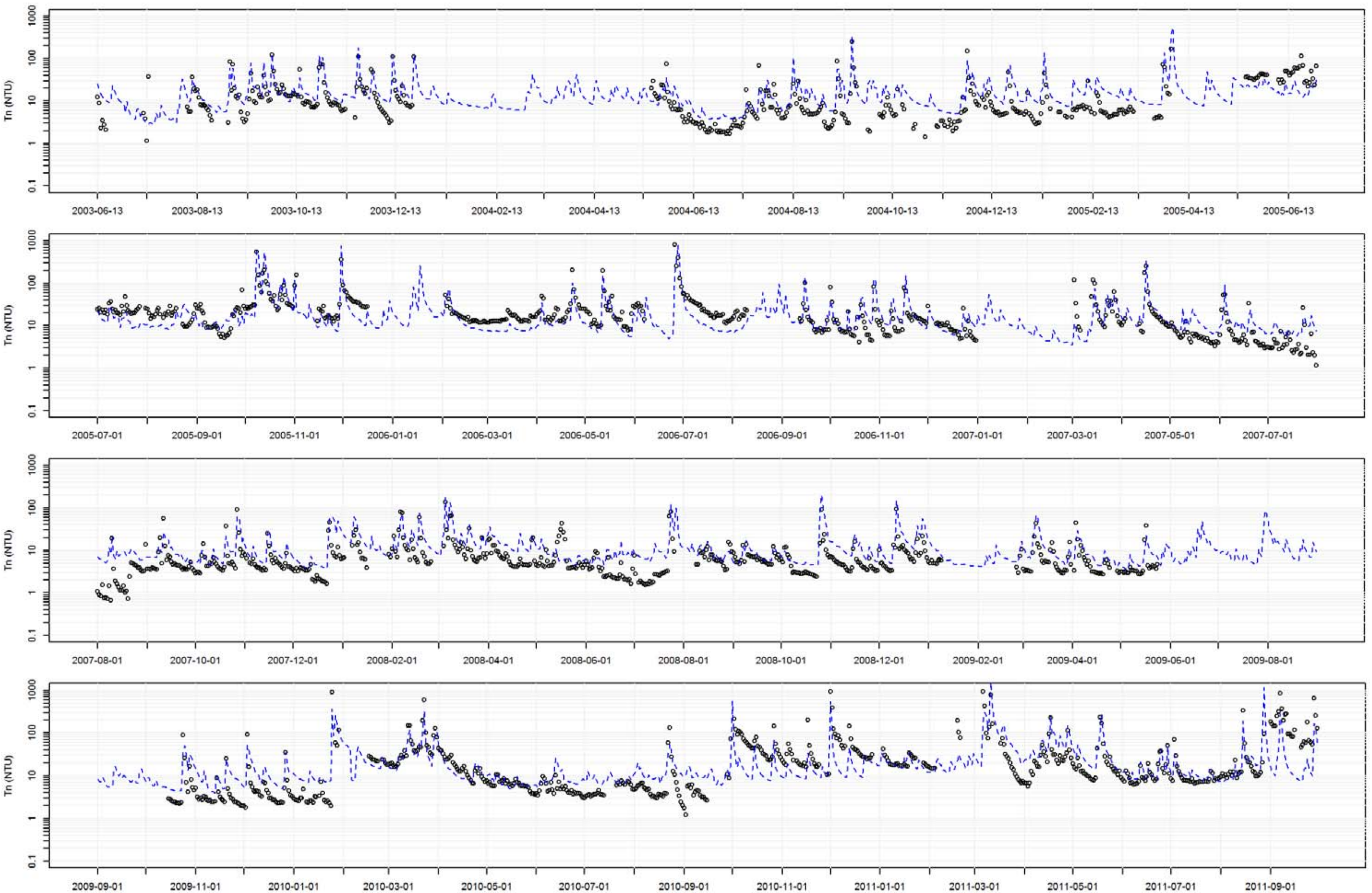
Linear Regression Rating Curve Fit

$$Y(t) = \sum_{i=1}^{c} \beta_i X_i(t) + \mu + \epsilon(t)$$

Predictor (*X*)	Rating Curve Coefficient (*β*)
Intercept (μ)	3.746
log(Watershed Q)	-1.082
log(Watershed Q)^2	0.129
Hysteresis	0.581
log(Cens. Shand. Tn)	0.620
log(Cens. Shand. Load)	-0.168

- R^2 = 0.55
- AIC = 4857.40

Fitted Rating Curve vs. Observations



Selecting a Time Series Model Based on Rule Curve Error Characteristics

- Sample autocorrelation function (ACF)
  - Computes the sample autocorrelation of time series points *k* units apart
- Sample partial autocorrelation function (PACF)
  - Partial autocorrelation (autocorrelation of ACF residuals) at *k* lags

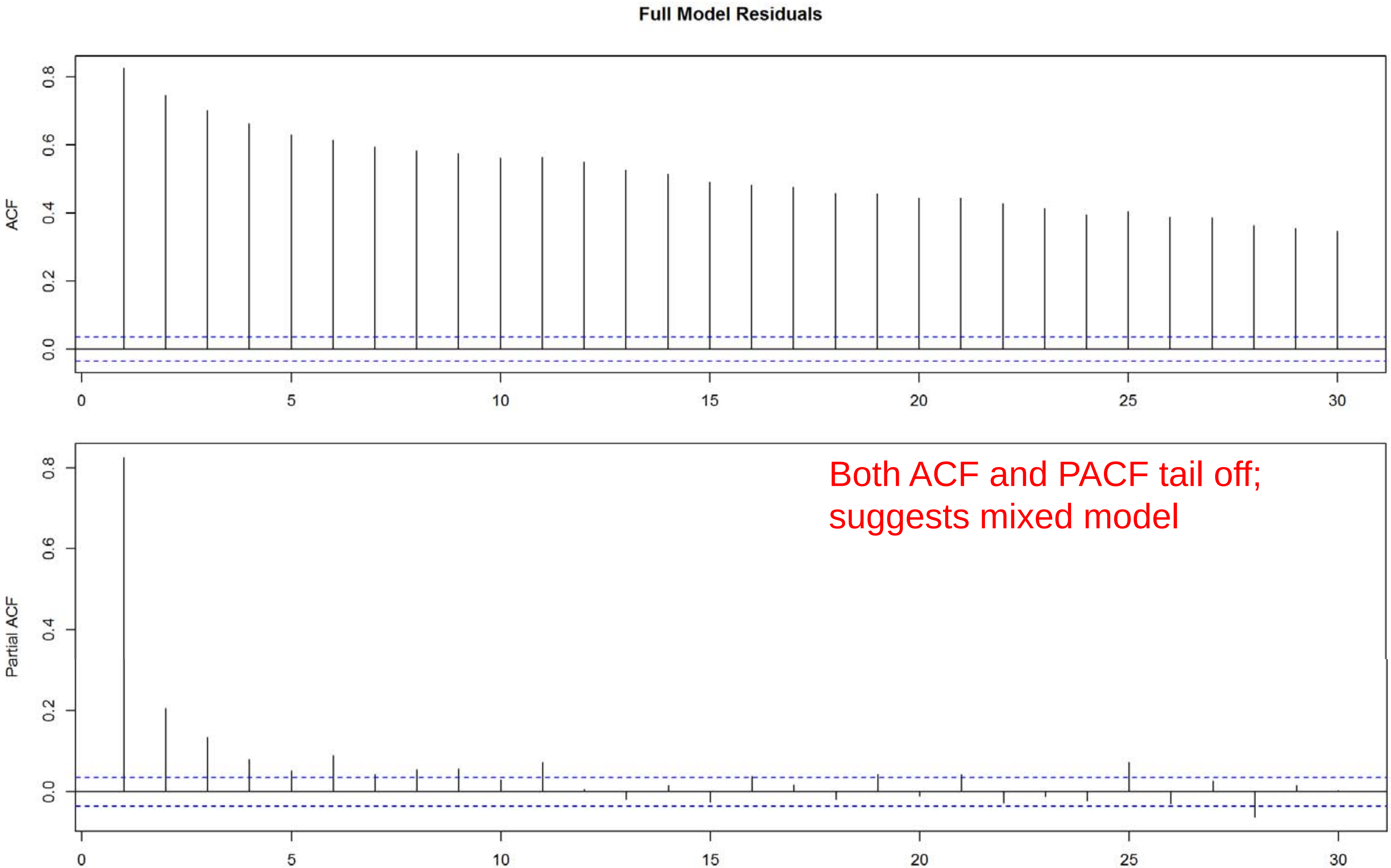
	AR(p)	**MA(q)**	**ARMA(p, q)**
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

*Table from Cryer & Chan, 2008

- Extended autocorrelation function (EACF)
  - Structure shows order or mixed ARMA model
  - Diagonal in matrix below reveals AR 1, MA 2 model – ARMA(1,2)

AR/MA	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	x	x	x	x	x	x	x	x	x
1	x	x	o	o	o	o	o	o	o
2	x	x	x	o	o	o	o	o	o
3	x	x	x	x	o	o	o	o	o
4	x	x	x	x	x	o	o	o	o
5	x	x	x	x	x	x	o	o	o
6	x	x	x	x	x	x	x	o	o
7	x	x	x	x	x	x	x	x	o

Rule Curve Error ACF and PACF



Rule Curve EACF

AR/MA Lag	0	1	2	3	4	5	6	7
0	x	x	x	x	x	x	x	x
1	x	x	0	0	0	0	0	0
2	x	x	x	0	0	0	0	0
3	x	x	x	x	x	0	0	0
4	x	x	x	0	0	0	0	0
5	x	x	x	x	0	0	0	0
6	0	x	x	x	x	0	0	0
7	x	x	x	x	x	0	0	0

- EACF's in real life never as clear as theoretical examples, must use some judgment
- Diagonal like structure exists with vertex at AR 1, MA 2

ARMA(1, 2) Error Model Fit

Linear Regression Part: $$Y(t) = \sum_{i=1}^{c} \beta_i X_i(t) + \mu + \epsilon(t)$$

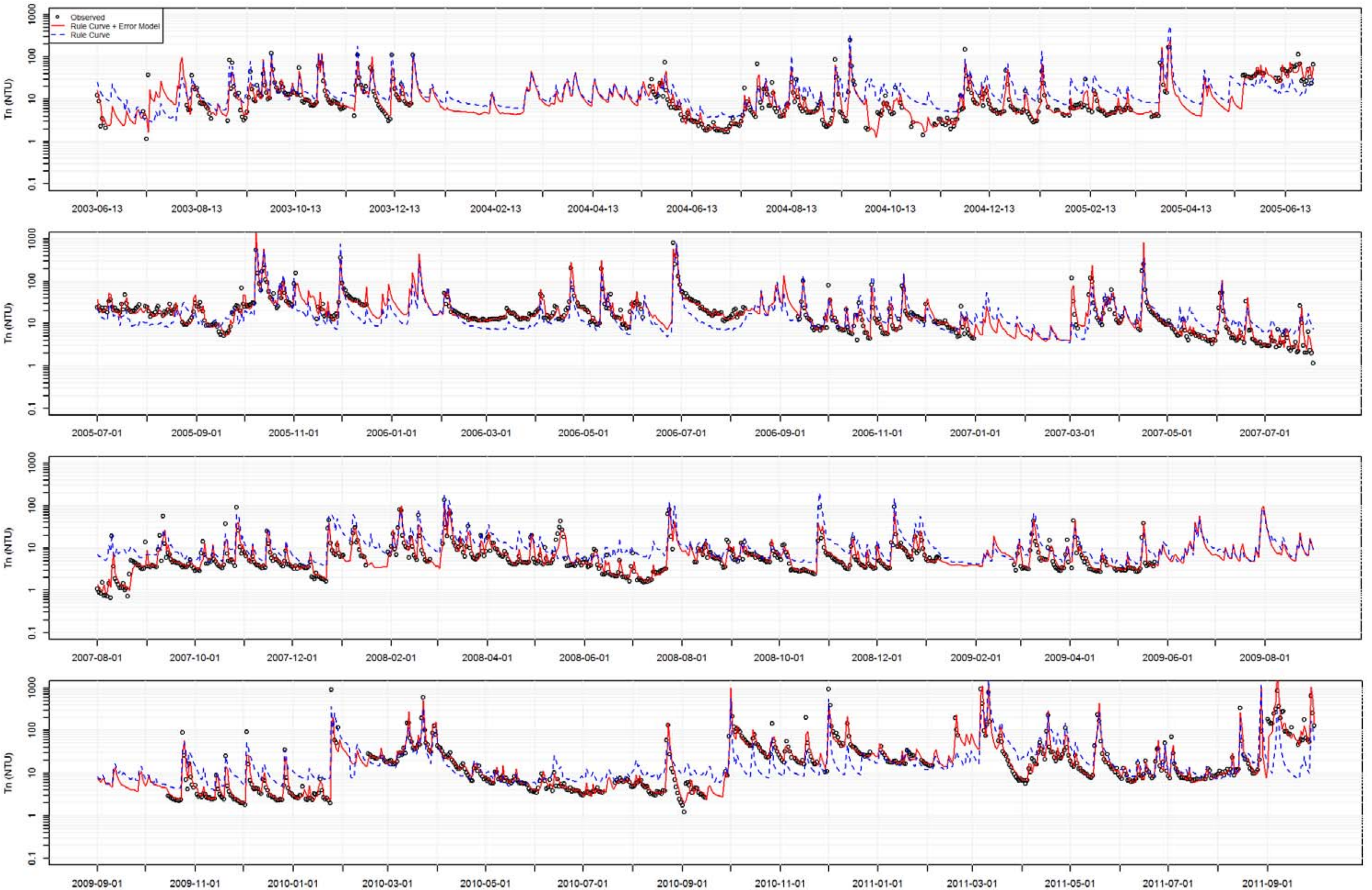
ARMA Errors: $$\epsilon(t) = \sum_{i=1}^{p} \phi_i[\epsilon(t-i) - \mu] + \sum_{i=1}^{q} \theta_i[\eta(t-i) - \mu]$$

Coefficients:

	ar1 (φ_1)	ma1 (ϑ_1)	ma2 (ϑ_2)	Mean (μ)	log(Watershed Q)	log(Watershed Q)^2	Hysteresis	log(Cens Shand tn)	log(Cens Shand Load)
Estimate	0.978	-0.290	-0.155	0.189	-0.130	0.069	0.565	-0.0809	0.0756

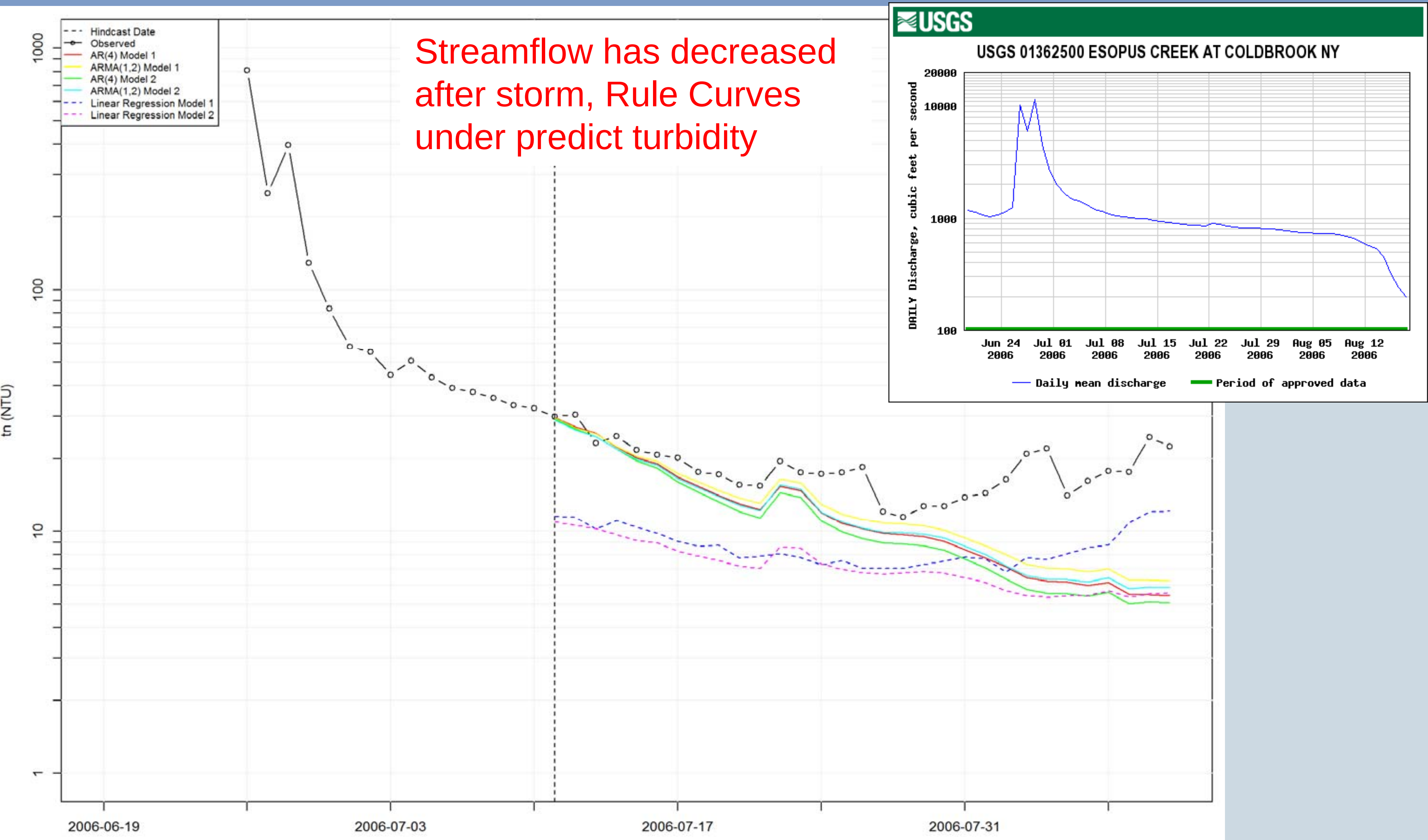
- AIC of Rule Curve + Error Model (1410.42) much smaller than Rule Curve alone (4857.40)

Fitted Models vs. Observations



Model Validation Examples:
Tail End of Turbidity Event, streamflow has regressed

Streamflow has decreased after storm, Rule Curves under predict turbidity



Model Validation Examples:
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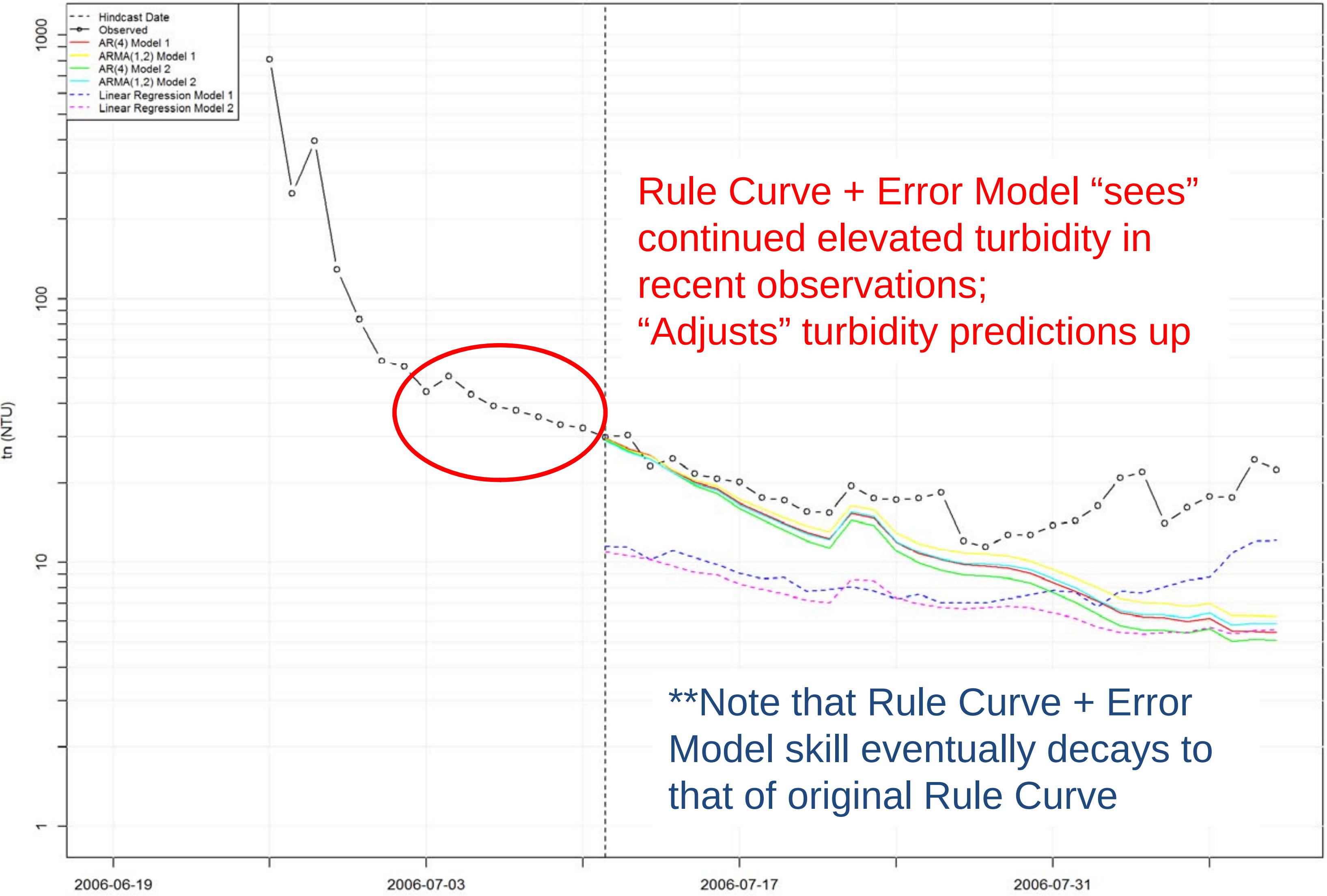
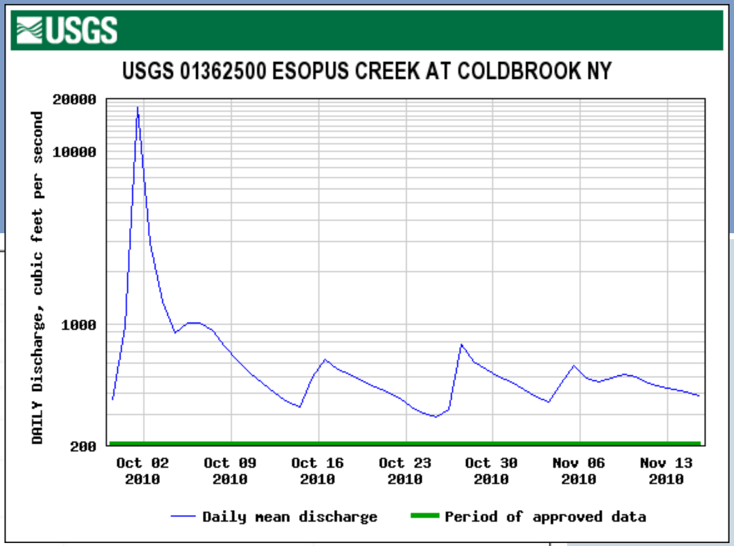
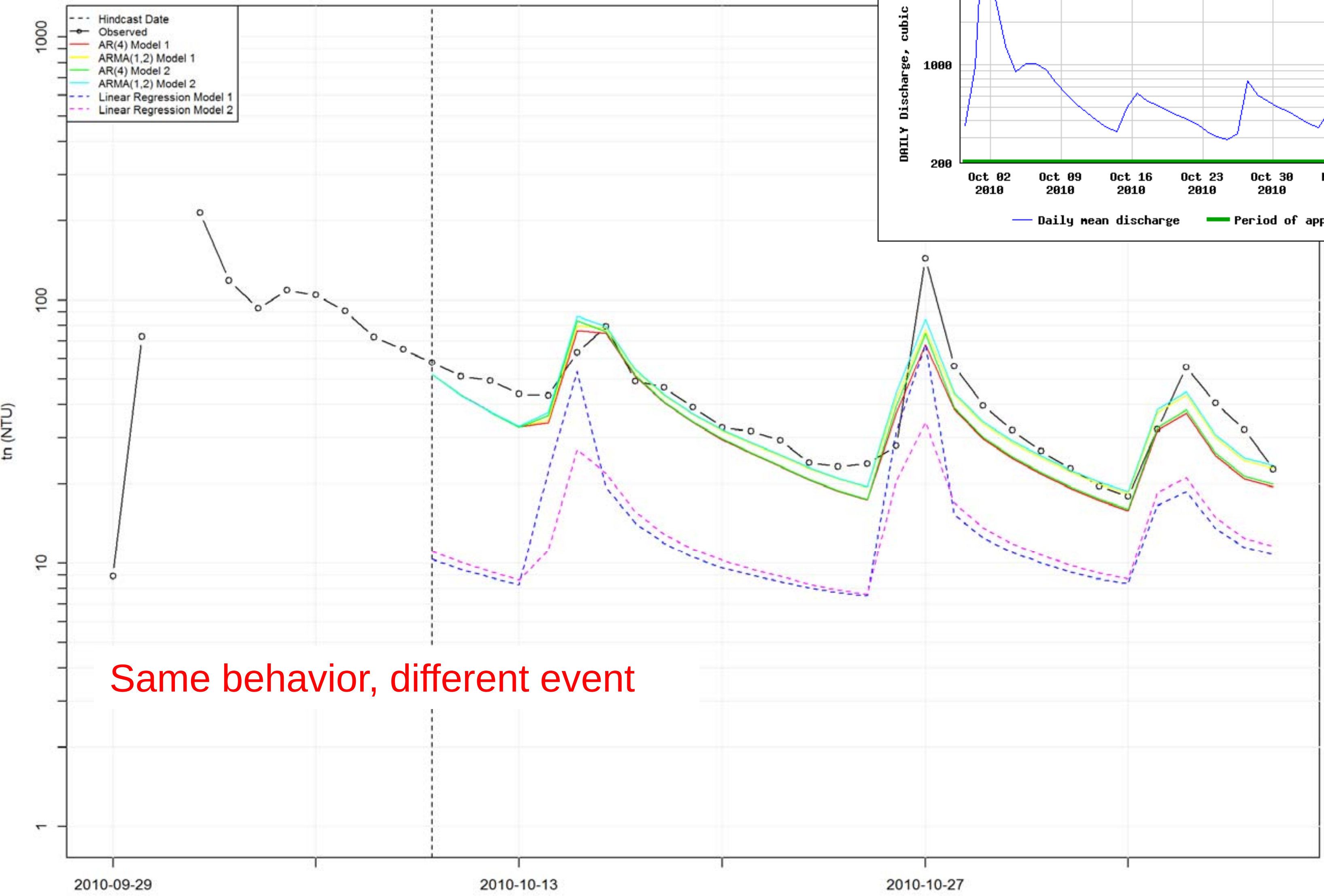


Rule Curve + Error Model “sees” continued elevated turbidity in recent observations;
“Adjusts” turbidity predictions up

**Note that Rule Curve + Error Model skill eventually decays to that of original Rule Curve

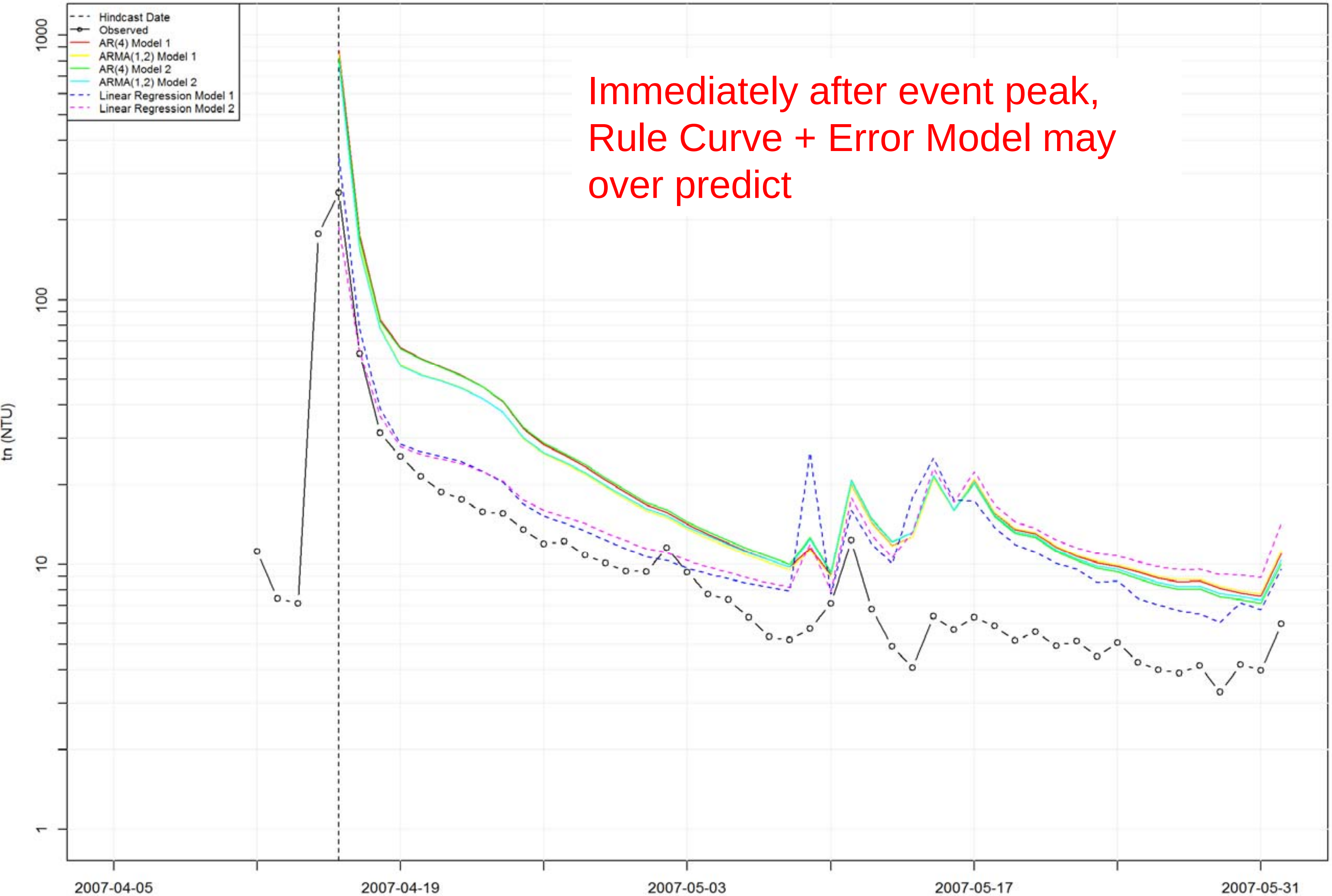
Model Validation Examples: Tail End of Turbidity Event, streamflow has regressed





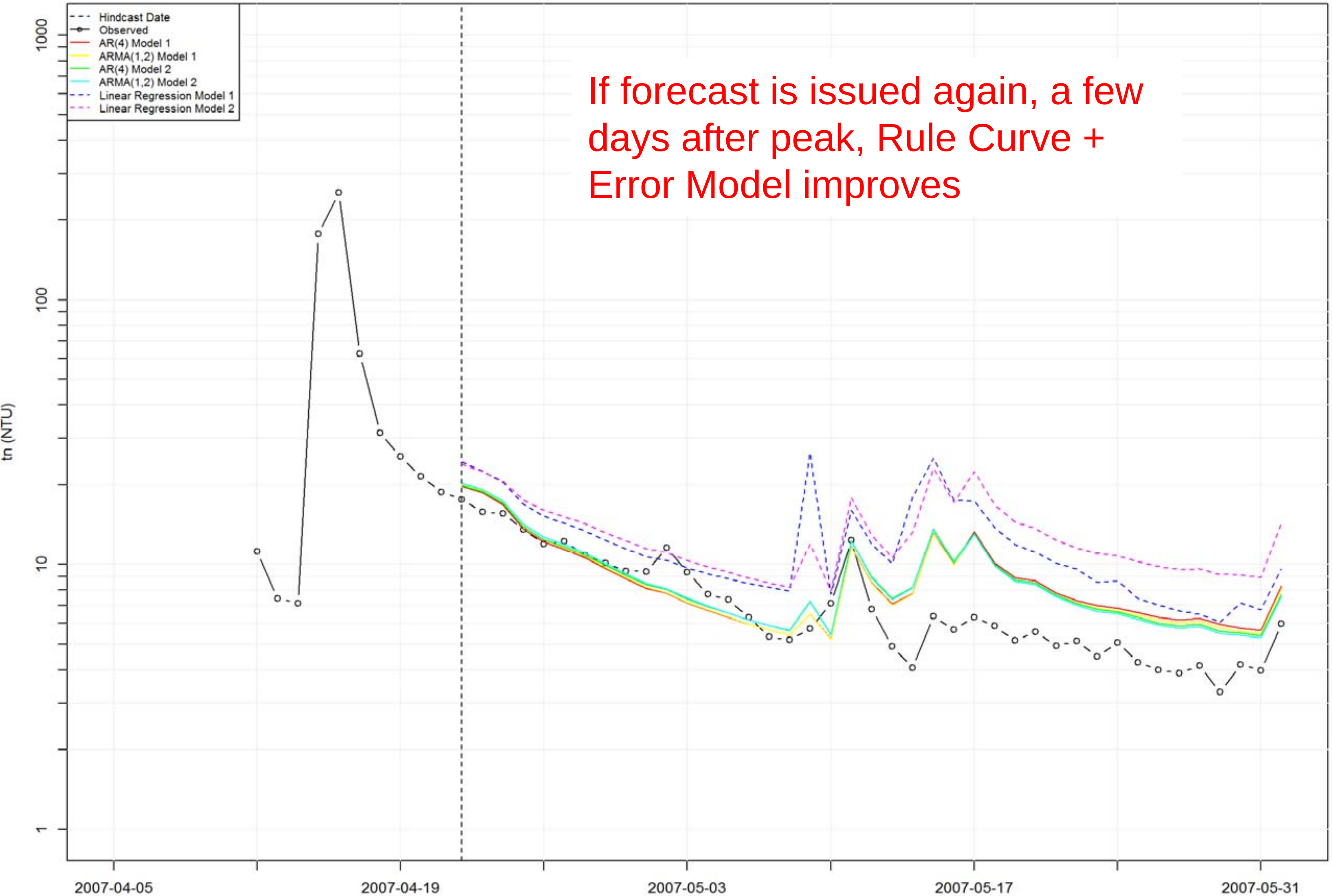
Same behavior, different event

Model Validation Examples: Predictions Immediately after Event Peak



Immediately after event peak, Rule Curve + Error Model may over predict

Model Validation Examples: Predictions Immediately after Event Peak



If forecast is issued again, a few days after peak, Rule Curve + Error Model improves

Conclusions & Next Steps

- Adding a time series error model to turbidity rating curves can increase short term skill of turbidity predictions
- Skill is best during falling limb of turbidity event
  - Largest improvements over traditional rule curves when flow has significantly regressed but ambient turbidity remains high
- Very data intensive!
  - Requires relatively long and continuous (few gaps!) historical record to calibrate/fit autoregressive models
  - Requires active, near real real-time turbidity monitoring and streamflow forecasts to run operationally
- Currently being integrated and tested in NYC's Operations Support Tool (OST) to improve turbidity control operations at Ashokan

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